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Abstract. This paper introduces an efficient diagnosis algorithm for regular structures (EDARS). The algorithm provides a diagnosis which is correct, but possibly incomplete, if the cardinality of the actual fault set is below a “syndrome-dependent bound”, asserted by the algorithm itself. The time complexity of EDARS is $O(nt)$ when executed on t -regular structures of size n .

The correctness and the degree of completeness of the diagnosis provided by EDARS, were evaluated by means of simulation. Grids, hypercubes and Cube-Connected Cycles (CCC) structures were considered. Simulation results with grid structures showed a strong influence of structure degree over diagnosis performance. Furthermore, comparisons of simulation results obtained with grids, hypercubes and CCCs of same size and degree, showed that diameter and bisection width also appear to influence the performance of EDARS, particularly with respect to diagnosis completeness.

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1 Introduction

System-level diagnosis, which was introduced by Preparata, Metze and Chien [1], aims at diagnosing systems composed by units (usually processors) connected by point-to-point bidirectional links. A system S is represented by the *system graph* $G=(N,E)$, an undirected graph where node set N represents the units and edge set E represents the links. Edge (u,v) exists if and only if units u and v are interconnected. The cardinality¹ $n=\#N$ is called the size of the system. If edge (u,v) exists, units u and v are said to be *adjacent*, denoted $u \leftrightarrow v$.

Diagnosis is based on tests between units. The *testing* unit u provides a test sequence as input to the *tested* unit v , which returns an output sequence to u . The testing unit compares the actual and the expected output sequences and provides a binary test outcome (0 if the actual and the expected results match, 1 otherwise).

The PMC model assumes that tests of faulty units performed by non-faulty units always return 1 (that is, the test has perfect coverage), while the test outcome of the tests performed by faulty units is arbitrary. This invalidation rule is shown in Table 1. Similar diagnostic models assume different invalidation rule [2] or are based on comparisons between units [3]. In the PMC model, every test requires a bidirectional link between the testing and the tested unit. In this paper it is assumed that tests are reciprocal and that any two units u,v with $(u,v) \in E$ test each other.

In the following, notation $u \xrightarrow{v}$ denotes the test of unit v performed by units u with outcome

Table 1. Invalidation rule of the PMC model.

Testing unit	Tested unit	Test outcome
Fault-free	Fault-free	0
Fault-free	Faulty	1
Faulty	Fault-free	0 or 1
Faulty	Faulty	0 or 1

¹ Throughout this paper $\#X$ denotes the cardinality of set X , for any X .

$\gamma \in \{0,1\}$. Notation $u \xrightarrow{\delta \gamma} v$ denotes both the test of unit v performed by u with outcome γ and the test of unit u performed by unit v with outcome δ . Given a set N_f of faulty units (*actual fault set*), the set of all test outcomes is called *syndrome*, denoted σ .

The syndrome is collected by an external, reliable diagnoser and it is decoded by a *diagnosis algorithm*. This algorithm provides a *diagnosis* of the system by partitioning set N into subsets F of units declared faulty, subset K of units declared non-faulty, and subset S of *suspect* units. Given a syndrome σ , the diagnosis is said to be *correct* if $F \subseteq N_f$ and $K \subseteq N - N_f$. The diagnosis is said to be *complete* if $S = \emptyset$.

A system is said *one-step t_0 -diagnosable* if correct and complete diagnosis is always possible for all fault sets N_f with $\#N_f \leq t_0$. The value of t_0 , called the *diagnosability* of the system, is limited above by the minimum number of the tests undergone by units in the system. Under the hypothesis of reciprocal tests, this equals the minimum of the node degrees in G [1,4]. A general one-step diagnosis algorithm is reported in [5].

As an alternative to the preceding “deterministic” approach, Scheinerman [6] considered random graphs (i. e., graphs in which every test link exists with probability p) and showed that correct and complete diagnosis can be obtained with probability approaching 1 as $n \rightarrow \infty$ if the average number of links per unit is slightly above $\log n$. Blough et al. [7] reinforced this result by proving that $\log n + c$ test links per unit, where c is a small constant, are necessary and sufficient to achieve asymptotically correct diagnosis.

The preceding results are of relatively little value in the perspective applications of this theory, i. e. the diagnosis of massive parallel processing systems and the “wafer-scale” VLSI testing [8]. In both applications the system graph is t -regular (i.e. every unit has the same degree t), and the node degree is (or it is upper-bounded by) a small constant, with the exception of some massive parallel systems where the interconnection structure is a hypercube.

Increasing the degree, i. e. the number of links, would imply increasing costs and, in the case of application to wafer-scale VLSI testing, would also conflict with technological constraints. For this reason, the theory of system-level diagnosis should pursue the best possible trade-off between diagnosis performance and cost of the interconnection structure.

Along this trend of research, Somani and Agarwal [9], LaForge et al. [10], and Huang et al. [11] presented probabilistic algorithms which apply to regular or quasi-regular systems. The proposed algorithms provide diagnoses which are complete but possibly incorrect, although the probability of incorrect diagnosis, which was studied analytically and/or by means of simulation, is relatively small.

Another approach [12] is based on algorithms which provide correct, although possibly incomplete, diagnosis. Diagnosis correctness is guaranteed by a deterministic bound or it is asserted by the diagnosis algorithm itself [13,14,15].

The diagnosis algorithm for regular structures presented and evaluated in this paper, is a further development of the latter approach. The algorithm has time complexity $O(nt)$ when executed on t -regular structures of size n . The performance of the algorithm, in terms of diagnosis correctness and completeness, was evaluated by means of simulation over two-dimensional grids, hypercubes and Cube-Connected Cycles (CCC). Simulation studies showed that the performance of the algorithm is sensible to the degree of the structure. They also provided evidence that the diagnosis completeness is affected by other structure parameters [16] such as diameter (defined as the maximum of the minimum distances between nodes) and bisection width (defined as the cardinality of the minimum number of links, whose removal partitions a graph into two subgraphs with equal cardinality). As far as the authors know, the influence of diameter and bisection width on diagnosis completeness was never measured before.

The grids, hypercubes and CCCs structures are defined as follows.

A grid structure of size $n=L^2$, where L is a positive, even integer, is composed by n units arranged in L columns and L rows. Each unit is indexed by a pair (x,y) of integers, with $x=0\dots L-1$ and $y=0\dots L-1$. Hereafter unit indexed by (x,y) will be denoted u_{xy} . Units are connected to a constant number of neighbors, according to the rules specified below. Depending on the number of neighbors (3, 4, 6 or 8), grids considered in this paper will be called *triangular*, *square*, *hexagonal* or *octagonal*. The units are connected according to the following rules:

- In *triangular grids*, denoted G_3 , unit u_{xy} is connected to units indexed by:
 - $(x,(y\pm 1) \bmod L)$;
 - $((x+1) \bmod L,y)$ if x and y are both even or both odd, and to $((x-1) \bmod L,y)$ otherwise.
- In *square grids*, denoted G_4 , unit u_{xy} is connected to units indexed by $(x,(y\pm 1) \bmod L)$ and $((x\pm 1) \bmod L,y)$.
- In *hexagonal grids*, denoted G_6 , unit u_{xy} is connected to units indexed by:
 - $(x,(y\pm 1) \bmod L)$ and $((x\pm 1) \bmod L,y)$;
 - $((x\pm 1) \bmod L, (y+1) \bmod L)$ if x is even, and to $((x\pm 1) \bmod L, (y-1) \bmod L)$ otherwise.
- In *octagonal grids*, denoted G_8 , unit u_{xy} is connected to units indexed by $(x,(y\pm 1) \bmod L)$, $((x\pm 1) \bmod L,y)$ and $((x\pm 1) \bmod L, (y\pm 1) \bmod L)$;

Grids G_3 , G_4 , G_6 and G_8 are 3, 4, 6 and 8-regular structures, due to the wrap-around links crossing the border: for this reason they are also called toroidal grids. Examples of grids of size 16 are shown in Figure 1.

Non-toroidal grids NG_3 , NG_4 , NG_6 and NG_8 of size n may be derived from G_3 , G_4 , G_6 and G_8 of the same size by removing the wrap-around links crossing the border. This implies that the degree of units lying on the border is smaller than the degree of internal units: hence, non-toroidal grids are quasi-regular structures. In this paper, consideration will be limited to toroidal grids.

An hypercube structure of dimension d , denoted H_d , is composed by $n=2^d$ units. Every unit u is labeled with a d -digits binary number denoted $lab(u)$. Units are connected based on the Hamming distance of their labels, denoted d_H : edge (u,v) exists if and only if $d_H(lab(u),lab(v))=1$.

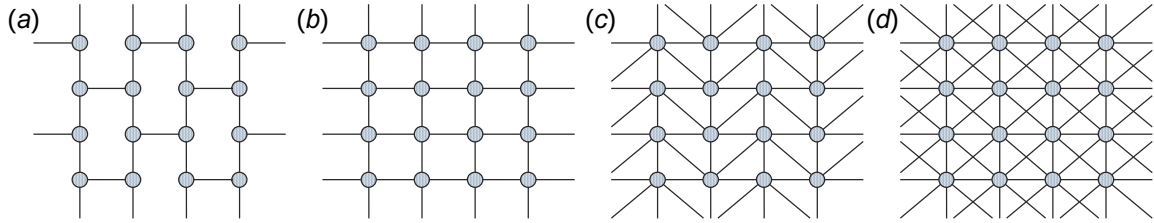


Figure 1. Grids G_3 (a), G_4 (b), G_6 (c), and G_8 (d) with $L=4$.

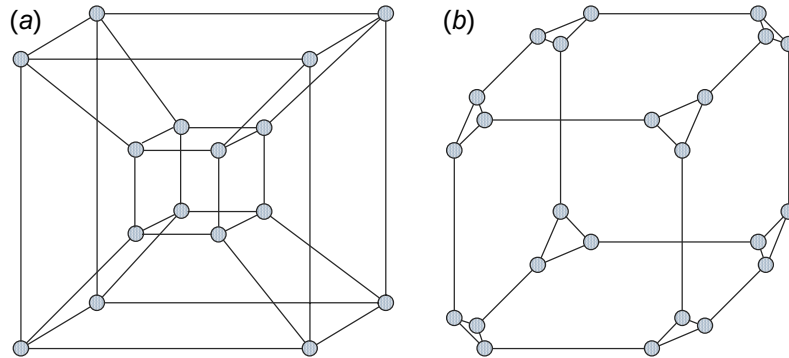


Figure 2. Hypercube with $d=4$ (a), and CCC with $d=3$ (b).

The hypercube of dimension $d=4$ is shown in Figure 2 (a). It is immediate that hypercubes are d -regular structures, and the degree $d=\log n$ increases with the size.

A Cube-Connected Cycles (CCC) structure of dimension d , denoted CCC_d , is composed by 2^d cycles of units, each comprising d units. Each unit is labeled with a pair (c, l) , $0 \leq c < 2^d$ and $0 \leq l < d$, where c is a d -digits binary number identifying a cycle and l is an integer identifying a unit within the cycle. Unit (c, l) is connected to units $(c, l \pm 1 \bmod d)$ and to unit $(c \oplus 2^l, l)$, where $\oplus$ denotes the bitwise exclusive-or operator. Every unit in the CCC has degree 3, hence the CCC is a 3-regular structure. CCC_3 is depicted in Figure 2 (b).

The rest of the paper is organized as follow. The EDARS algorithm is introduced in Section 2. The syndrome-dependent and the syndrome-independent bounds are introduced in Sections 3 and 4, respectively, and the issues related to the completeness of the diagnosis are discussed in Section 5. Section 6 presents the simulation experiments. Conclusions are drawn in Section 7.

2 An Efficient Diagnosis Algorithm for Regular Structures

In this section we introduce the diagnosis algorithm EDARS (Efficient Diagnosis Algorithm for Regular Structures), based on the PMC model. The following definitions are exploited in the description of the algorithm.

Given any syndrome σ , an *aggregate* A is a connected component of the system graph of node set $A \subseteq N$. The cardinality of A , denoted $\#A$, is called the *area* of A . The set $B(A) = \{u \notin A \mid \exists v \in A, u \leftrightarrow v\}$ is called the boundary of A . Hereafter we will use the word aggregate to mean both a connected subgrid A and its node set A . An aggregate A is a *Z-aggregate* if $u \xrightarrow{0 \ 0} v$ for every pair u, v with $u, v \in A$, $u \leftrightarrow v$.

From the invalidation rule of the PMC model, the following properties are immediate:

- (a) if $u \xrightarrow{1 \ 0} v$, then u is faulty;
- (b) if u is known to be faulty, then any unit $v \in A_0(u)$ is also faulty;
- (c) if $u \xrightarrow{1 \ 1} v$, then at least one unit between u and v is faulty;
- (d) if $u \xrightarrow{0 \ 0} v$, then u and v are both faulty or both non-faulty.

The following definitions have been introduced in [5]:

A directed path from u to v consisting of 0-labeled edges, is denoted by $u \Rightarrow_0 v$. Given a syndrome σ , any unit $u \in N$, and any subset $X \subseteq N$, sets $\Delta_1(u)$, $\Delta_1(X)$, $D_0(u)$, $D_0(X)$, $A_0(u)$, $A_0(X)$ and $L(u)$ are defined as follows:

- $\Delta_1(u) = \{v \in N: u \xrightarrow{1} v \text{ or } v \xrightarrow{1} u\}$, $\Delta_1(X) = \bigcup_{u \in X} \Delta_1(u)$ are the *disagreement sets* of u and X , respectively;
- $D_0(u) = \{v \in N: u \Rightarrow_0 v\} \cup \{u\}$, $D_0(X) = \bigcup_{u \in X} D_0(u)$ are the *zero descendant sets* of u and X , respectively;
- $A_0(u) = \{v \in N: v \Rightarrow_0 u\}$, $A_0(X) = \bigcup_{u \in X} A_0(u)$ are the *zero ancestor sets* of u and X , respectively;
- $L(u) = \Delta_1(D_0(u)) \cup A_0(\Delta_1(D_0(u)))$ is the *implied faulty set* of u .

Informally, the implied faulty set of a unit u is defined as the set of units that must be faulty under the assumption that u is non-faulty.

Given a syndrome σ , EDARS is divided into three subsequent steps.

The first step (*Local Diagnosis*) performs a preliminary classification of units, partitioning N into sets F , D and Z . Set F contains faulty units identified by exploiting properties (a) and (b). Units in set D (*dual units*) are defined in disjoint pairs with the property that, for every pair, each unit accuses the other of being faulty. From Property (c), at least one unit in every pair must be faulty. The remaining units are assigned to set Z (*zero-units*). Adjacent zero-units must test each other with outcome 0, since otherwise the units should have been put into set F or D . By Property (d), they must be in the same state.

The second step (*Fault-Free Core Identification*) partitions the subgraph Z , of node set Z , into Z -aggregates. It is immediate that all units in a Z -aggregate are in the same state. The *Fault-Free Core (FFC)* is defined as the union set of the Z -aggregates of maximum cardinality. Under the hypotheses stated in Theorem 1, the *FFC* is non-empty and actually fault-free. In this step, the algorithm also asserts the syndrome-dependent bound T_σ , with the property that the diagnosis returned by EDARS is correct if the actual number of faulty units is less than T_σ .

In Step 3 (*Augmentation*) the *FFC* is recursively augmented with units in $D_0(FFC)$, which are non-faulty under the invalidation rule of Table 1. Similarly, set F constructed in Step 1 is augmented with units belonging to $\Delta_1(FFC)$ and to $A_0(FFC)$, which must be faulty under the rules of Table 1.

As shown in [15], the time complexity of EDARS is $O(nt)$. The algorithm is reported in Table 2.

Table 2. The EDARS algorithm.

```

{initialization: sets  $F$ ,  $D$ , and  $Z$  are empty}
   $F := \emptyset$ ;  $D := \emptyset$ ;  $Z := \emptyset$ ;
{Step one: Local Diagnosis}
  {identification of faulty units}
  While  $\exists u \in N - F, \exists v \in N: u \xrightarrow{1 \ 0} v$  Do  $F := F \cup \{u\}$  EndWhile { $u$  is faulty by Property (a) }
   $F := F \cup A_0(F)$  { $F$  is augmented according to Property (b)}
  {identification of dual units}
  While  $\exists u, v \in N - (F \cup D): u \xrightarrow{1 \ 1} v$  Do  $D := D \cup \{u, v\}$  EndWhile
  {identification of set  $Z$  of zero-units}
   $Z := N - (D \cup F)$ ;
{Step Two: Fault-Free Core Identification}
   $Q := \text{EmptyQueue}$ ;  $h := 1$ ; { $Z$ -aggregate count}
  While  $\exists u \in Z$  do
     $Z := Z - \{u\}$ ;  $Z_h := \{u\}$ ;  $h := h + 1$ ; insert( $u, Q$ );
    { $u$  is removed from  $Z$  and inserted into  $Q$ }
    While NonEmpty( $Q$ ) Do
       $u := \text{remove}(Q)$ ;
      for each  $v \in Z: u \leftrightarrow v$  Do  $Z := Z - \{v\}$ ;  $Z_h := Z_h \cup \{v\}$ ; insert( $v, Q$ ) EndDo
    EndWhile
  EndWhile;
   $\alpha := \text{MaxCard}(Z_1, \dots, Z_h)$ ;
  {given sets  $S_1, \dots, S_n$  MaxCard( $S_1, \dots, S_n$ ) returns the maximum of  $\#S_1, \dots, \#S_n$ }
   $FFC := \bigcup_{\#Z_i = \alpha} Z_i$  {set  $FFC$  is the Fault-Free Core}
   $T_\sigma := \alpha + \#F + \#D / 2$ ; {syndrome-dependent bound}
Assertion: the diagnosis is correct if  $\alpha > 0$  and  $\#N_f < T_\sigma$ ;
{Step Three: Augmentation}
   $FFC := FFC \cup D_0(FFC)$ ; {the zero descendants of the  $FFC$  are non faulty}
   $F := F \cup \Delta_1(FFC)$ ; {the units in the disagreement set of the  $FFC$  are faulty}
   $F := F \cup A_0(F)$ ; {zero ancestors of faulty units are faulty}
{Diagnosis and Validation}
   $S := N - (FFC \cup F)$ ; { $S$  is the set of suspect units}
return( $FFC, F, S, T_\sigma$ );

```

It should be observed that the cardinality of set F constructed in Step 1 could be increased by determining the implied faulty set of every unit and by declaring faulty those units which are in their own implied faulty set. It is easily seen that set F constructed by the algorithm of Table 2 is a subset of the set which would result from the preceding rule. Similarly, it should be observed that the rule used in Table 2 to construct set D is a matching on the subgraph induced by units which test each other with outcome 1, and that $\#D$ could be maximized (at the expense of the Z-aggregates cardinalities) by using a maximum matching algorithm. However, this would increase the time complexity of the algorithm and, as evidenced by simulation experiments, the advantages, if any, would be negligible [15].

3 Diagnosis Correctness: the Syndrome-Dependent Bound

Given any syndrome σ , the diagnosis is correct if there exists at least one Z-aggregate (that is $\alpha > 0$) and every Z-aggregate of area α is fault-free. Under these conditions the FFC defined in Step 2 is non-empty and actually fault-free. Under the same conditions, Step 3 augments set FFC with units which are actually non-faulty and set F with units which are actually faulty.

Given any syndrome σ , the following theorem relates the diagnosis correctness to the *syndrome-dependent bound* T_σ asserted by the diagnosis algorithm in Step 2:

Theorem 1. *Given syndrome σ , the diagnosis returned by EDARS is correct provided $\alpha > 0$ and $\#N_f < T_\sigma$*

Proof. Recalling that set D is constructed by addition of disjoint pairs of units and that at least one unit in every pair must be faulty, the number of faults in the system is at least $\#F + \#D/2$ (where F is the set of faulty units constructed in the first step). If some Z-aggregate of area α , where α is the maximum of the Z-aggregate cardinalities, is not fault-free, this implies that the Fault-Free Core contains α faulty units, and the diagnosis is incorrect. However, this also implies that $\#N_f \geq \alpha + \#F + \#D/2 = T_\sigma$ thus contradicting the hypothesis $\#N_f < T_\sigma$. $\square$

Given some (unknown) fault set N_f and syndromes σ arising from N_f , assume that F is the set of units declared faulty by EDARS and that T_σ is the syndrome-dependent bound asserted by the algorithm itself. If the diagnosis is correct, must be $F \subseteq N_f$. From Theorem 1, the response of the algorithm can be relied upon as a correct diagnosis if $\#F < T_\sigma$ and it is firmly expected that the maximum cardinality of N_f be less than T_σ .

It should be kept in mind that bound T_σ depends on the cardinalities of sets constructed in steps 1 and 2 of EDARS (namely, $\#F$, $\#D$, and α). Such cardinalities are dependent on the current syndrome σ , which, in turn, depends on the actual fault set. Simulations experiments reported in Section 6 showed that the diagnosis validation based on T_σ is quite reliable, since the average of T_σ always resulted above $\#N_f$ as long as $\#N_f$ did not exceed $n/2$ (with the exception of some CCC structure). In most cases the difference between the average of T_σ and $\#N_f$ was comfortably large and the standard deviation was quite small.

4 Diagnosis Correctness: the Syndrome-Independent Bound

The *syndrome-independent bound*, denoted T , is defined as the minimum of T_σ over set Σ of all syndromes which give rise to at least one Z-aggregate. Contrary to T_σ bound T ensures correct diagnosis whenever the cardinality of the actual fault set is less than T , independently of the actual syndrome. Under this hypothesis, letting σ be any admissible syndrome of N_f , the correctness of the diagnosis is ensured by Theorem 1, since $T_\sigma \geq T$.

The syndrome-independent bounds for square grids, denoted T_4 , and for non-toroidal square grids, denoted NT_4 , were derived by a worst case analysis in [13] and [14]. The syndrome-independent bound for non-toroidal hexagonal and octagonal grids, denoted NT_6 and NT_8 , respectively, was derived by a similar analysis in [15]. The syndrome-independent bounds NT_6 and NT_8 may be taken as approximations (from below) of the bounds for G_6 and G_8 .

These bounds could not be expressed analytically; however, they could be tightly limited by the following expressions, from which it is seen that all bounds are $\Theta(n^{2/3})$:

- T_4 : $\frac{3}{2} \left(\frac{17}{18} n \right)^{2/3} - \frac{170}{27} \left(\frac{10}{9} n \right)^{1/3} + 5 < T_4 < \frac{3}{2} \left(\frac{17}{18} n \right)^{2/3} + \frac{1}{2}$;
- NT_4 : $\frac{3}{2} \left(\frac{17}{18} (n+2\sqrt{n}-\frac{4}{3}) \right)^{2/3} - \frac{170}{27} \left(\frac{10}{9} (n+2\sqrt{n}-\frac{4}{3}) \right)^{1/3} - \sqrt{n} + \frac{23}{3} \leq NT_4 \leq \frac{3}{2} \left(\frac{17}{18} (n+2\sqrt{n}-\frac{4}{3}) \right)^{2/3} - \sqrt{n} + \frac{7}{6}$;
- NT_6 : $\left(\frac{9}{4} (n+2\sqrt{n}) \right)^{2/3} - 4(n+2\sqrt{n})^{1/3} + \frac{35}{12} - \sqrt{n} < NT_6 < \left(\frac{9}{4} (n+2\sqrt{n}) \right)^{2/3} - \sqrt{n} + \frac{1}{4}$;
- NT_8 : $\left(\frac{27}{4} \right)^{1/3} (n+2\sqrt{n}+1)^{2/3} - \frac{16}{3} \left(\frac{3}{4} (n+2\sqrt{n}+1) \right)^{1/3} - \sqrt{n} + \frac{23}{18} < NT_8 < 2(n+2\sqrt{n}+1)^{2/3} - \sqrt{n} - \frac{1}{2}$;

Some values of T_4 , NT_4 , NT_6 and NT_8 for grids of different degrees and sizes are reported in Table 3.

Table 3. Values of T_4 , NT_4 , NT_6 and NT_8 for some grid sizes.

n	T_4	NT_4	NT_6	NT_8
64	16	10	16	18
1024	128	101	129	145
16384	885	762	937	1044

5 Diagnosis Completeness

Given any syndrome σ and the partition of set N into subsets FFC , F and S provided by EDARS, the diagnosis is incomplete if $S \neq \emptyset$. Assume $S \neq \emptyset$ and consider the subgraph of G induced by S : it is immediate that $S \cap \Delta_1(FFC) = \emptyset$, $S \cap D_0(FFC) = \emptyset$, and $S \cap A_0(F) = \emptyset$. Let $S_1, \dots, S_h$, of node sets $S_1, \dots, S_h$, be the connected components of S , and consider the boundaries $B_1, \dots, B_h$ of aggregates $S_1, \dots, S_h$. It is immediate that, for every B_i , $i=1, \dots, h$, must be $B_i \cap FFC = \emptyset$, since otherwise would be $S \cap \Delta_1(FFC) \neq \emptyset$ or $S \cap D_0(FFC) \neq \emptyset$. Thus, all the units in B_i must be faulty. This means that $\# \bigcup_{i=1, \dots, h} B_i$ provides a lower bound to the number of faults needed to produce incomplete diagnosis with the given syndrome. A syndrome-independent lower bound could be found by taking the minimum of $\# \bigcup_{i=1, \dots, h} B_i$ over the set of all possible syndromes.

Given an arbitrary aggregate S of area β , lower bounds to the cardinality of its boundary were given in [13] for square grids and in [15] for hexagonal and octagonal grids. These bounds are as follows:

- $f_1(\beta) = 2 + 2\sqrt{2\beta - 1}$ in square grids;
- $f_2(\beta) = 3 + \sqrt{12\beta - 3}$ in hexagonal grids;
- $f_3(\beta) = 4 + 4\sqrt{\beta}$ in octagonal grids.

Since f_1 , f_2 and f_3 are increasing functions of β , the minimum of $\# \bigcup_{i=1, \dots, h} B_i$ occurs when $h=1$ and $\#S=1$, that is, there exists a unique S_i of area $\beta=1$. This means that the diagnosis provided by EDARS may be incomplete only if the number of faults is at least $f_1(1)=4$ in square grids, at least $f_2(1)=6$ in hexagonal grids and at least $f_3(1)=8$ in octagonal grids. Observe that these values coincide with the degrees t of the respective grids, which are also known to be upper bounds to the one-step diagnosability. Assuming at most t faults, EDARS could be trivially extended to diagnose as non-

faulty the (unique) unit in S . This means that EDARS may also be regarded as an efficient algorithm for one-step diagnosis of grids.

The preceding worst case analysis is very pessimistic. In fact, the probability that $\#N_f$ faults distributed over set N yield at least one boundary of faulty units circumscribing a non-empty aggregate may be very small. An evaluation of such probability, which required meticulous analysis of the interconnection structure, was provided in [12] and [10] for the case of square grids. Extending this analysis to other regular structures could prove to be very difficult while providing limited results. The research reported in this paper used simulation to estimate the probability of incomplete diagnoses and the degree of incompleteness for a wide range of regular structures. One important goal of simulation was to gain an insight of the parameters of the interconnection structure which influence diagnosis completeness. One such parameter is clearly the degree; however, simple reasonings suggest that diameter and bisection width may also play some role.

Consider an arbitrary unit u not yet diagnosed as either non-faulty or faulty as the second step of EDARS is completed. This unit may be diagnosed during the Augmentation step, provided it is reached by either a chain of 0-descendants of the FFC or a chain of 0-ancestors of set F . With uniform distribution of faults, the probability of such chains decreases with their lengths. A small diameter of the interconnection structure appears to indicate that all chains connecting unit u to the FFC or to set F tend to be relatively short and, thus, relatively likely to occur. On the other hand, the probability of diagnosing u is also related to the number of distinct paths between u and units belonging to either FFC or F . In turn, this number is somehow related to the bisection width of the structure, and a larger bisection width could decrease the probability that the state of units remains unidentified.

As it will be reported in Section 6, the simulation results yielded by EDARS with structures of same degree but different diameter and/or bisection width, closely agree with the preceding conjecture.

6 Simulation Experiments

Extensive simulation experiments were conducted to the purpose of evaluating the performance of EDARS with the regular structures defined in Section 1. For every simulation experiment, parameters defining the structure and its size, the fault distribution and its parameters, and the number of faults to be distributed are given. Fault distribution may be uniform, in the same failure probability is assigned to every unit, or multivariate, in which a random number of faults are distributed uniformly around some “seeds”, also distributed uniformly. For the sake of brevity, all results reported in this paper refer to uniform distribution.

Once the faults have been distributed over the structure, the syndrome is generated according to the invalidation rule of the PMC model described in Table 1. The outcomes of tests performed by faulty units are determined randomly, assuming probability p , or q , for test outcome 1 if the tested unit is non-faulty, or faulty, respectively. Probabilities p and q are input parameters of the simulator.

Every simulation experiment consists of executing EDARS over a number of fault sets (sample), chosen to guarantee a target confidence interval⁽¹⁾ for all the output data of the experiment. The size of the sample was always at least 250, unless a larger size was required to achieve the target confidence interval with output data which were to be averaged over a subset of the sample (e.g., those fault sets which led to incomplete diagnosis).

The main data reported for every simulation experiment are:

- the percentage of fault sets which led to correct diagnosis (*%correct*);

⁽¹⁾ The confidence interval [17], calculated with precision r , is defined as the number i such that the probability $P(|E(T_o) - \mu(T_o)| \leq i)$ is greater than or equal to r , where $\mu(T_o)$ is the average of T_o calculated over the universe of all possible fault sets of given cardinality. In all the experiments performed r was set to 0.98.

- the average of the syndrome-dependent bound ($E(T_{\mathcal{D}})$);
- the percentage of fault sets which led to incomplete diagnosis ($\%incomplete$);
- the average number of units left unidentified by EDARS, considering only those fault sets which led to incomplete diagnosis ($E(\#N_d)$);
- The percentage of units left unidentified by EDARS considering the entire sample ($\%suspect$).

Additional parameters reported by the simulator are the average cardinalities of sets F and D constructed in the first step of EDARS, the average cardinality of the FFC constructed in the second step of EDARS, and the average numbers of non-faulty and faulty units left undiagnosed by EDARS, when diagnosis resulted incomplete. The confidence interval and the standard deviation are also returned along with every output parameter.

As an example, Table 4 displays some output data reported by simulation experiments with some structures of size $n=64$, namely G_3 , G_4 , G_6 and G_8 , hypercube H_6 and CCC_4 . For every structure, fault sets of cardinalities ranging from $0.1n$ to $0.5n$ were distributed uniformly over the node set, and the syndromes were generated assuming probabilities $p=0.5$ and $q=0.5$. The table reports the output parameters $\%incomplete$, $E(\#N_d)$, and $E(T_{\mathcal{D}})$. Parameter $\%correct$ is not reported since all diagnoses resulted correct. The confidence interval (ci) and the standard deviation (sqm) of $E(\#N_d)$ and $E(T_{\mathcal{D}})$ are also reported.

Table 4. Simulation results for structures of size $n=64$.

	$\#N_f$	$.1n$	$.2n$	$.3n$	$.4n$	$.5n$
G_3	$\%incomplete$	3.53	31.45	82.51	100.00	100.00
	$E(\#N_d)$, (ci , sqm)	1.11 (0.07, 0.35)	1.49 (0.11, 0.89)	4.78 (0.57, 4.99)	14.97 (0.9, 7.42)	21.08 (0.79, 5.99)
	$E(T_{\mathcal{D}})$, (ci , sqm)	63.18 (0.08, 0.79)	61.68 (0.22, 2.07)	56.71 (0.63, 5.97)	46 (0.72, 6.87)	42.53 (0.5, 4.77)
G_4	$\%incomplete$	0.20	4.10	22.81	73.75	96.90
	$E(\#N_d)$, (ci , sqm)	1 (0, 0)	1.04 (0.05, 0.24)	1.44 (0.12, 0.93)	3.11 (0.39, 3.38)	9.33 (0.69, 5.79)
	$E(T_{\mathcal{D}})$, (ci , sqm)	63.67 (0.06, 0.54)	63.16 (0.09, 0.88)	62.35 (0.14, 1.3)	59.21 (0.42, 4.03)	52.41 (0.63, 6)
G_6	$\%incomplete$	0.00	0.03	1.07	9.65	32.30
	$E(\#N_d)$, (ci , sqm)	0 (0, 0)	1 (0, 0)	1.1 (0.09, 0.3)	1.22 (0.06, 0.5)	1.71 (0.17, 1.43)
	$E(T_{\mathcal{D}})$, (ci , sqm)	63.94 (0.03, 0.24)	63.83 (0.04, 0.41)	63.68 (0.06, 0.55)	63.43 (0.08, 0.74)	62.99 (0.15, 1.42)
G_8	$\%incomplete$	0.00	0.00	0.03	1.13	9.15
	$E(\#N_d)$, (ci , sqm)	0 (0, 0)	0 (0, 0)	0 (0, 0)	1.24 (0.18, 0.61)	1.49 (0.11, 1.08)
	$E(T_{\mathcal{D}})$, (ci , sqm)	63.98 (0.02, 0.14)	63.96 (0.02, 0.21)	63.88 (0.04, 0.35)	63.83 (0.05, 0.47)	63.7 (0.06, 0.61)
H_6	$\%incomplete$	0.00	0.17	1.47	9.14	32.90
	$E(\#N_d)$, (ci , sqm)	0 (0, 0)	1 (0, 0)	1 (0, 0)	1.08 (0.04, 0.32)	1.42 (0.1, 0.8)
	$E(T_{\mathcal{D}})$, (ci , sqm)	63.94 (0.03, 0.25)	63.78 (0.05, 0.44)	63.68 (0.06, 0.54)	63.47 (0.08, 0.74)	63.03 (0.11, 1.05)
CCC_4	$\%incomplete$	3.37	33.47	88.97	100.00	100.00
	$E(\#N_d)$, (ci , sqm)	1.22 (0.12, 0.6)	2.19 (0.2, 1.68)	6.33 (0.65, 5.63)	18.47 (0.88, 7.23)	22.07 (0.69, 5.03)
	$E(T_{\mathcal{D}})$, (ci , sqm)	63.18 (0.09, 0.82)	61.19 (0.28, 2.66)	54.37 (0.71, 6.74)	43.26 (0.65, 6.17)	41.78 (0.44, 4.22)

6.1 Evaluation of EDARS with grids

Simulation experiments with grids covered sizes ranging from 64 to 16384, fault sets cardinalities ranging from $0.1n$ to $0.5n$, uniform and multivariate distributions, probabilities p and q ranging from 0.25 to 0.75 in steps of 0.25. Results are summarized in Figures 3 to 7 which, for the sake of brevity, report only results obtained with uniform distribution and with probabilities $p=q=0.5$. Results obtained with multivariate distribution and with different values of p and q were quite similar. Complete simulation results are reported in [15].

Referring to grids of size 64, 1024 and 16384, Figures 3 and 4 report $E(T_{\mathcal{D}})$ expressed as percentage of the size of the grids. It is seen that $E(T_{\mathcal{D}})$ is above (in the cases G_6 and G_8 , far above) the cardinality

of the actual fault set in the entire cardinality range covered by simulation. This means that the diagnosis of grids returned by EDARS is quite reliable if the expected number of faulty units is below $0.5n$. One interesting result displayed in the figures is that the value $E(T_\sigma)/n$ decreases as the size of the grid increases, indicating that the diagnosis does not converge toward asymptotic correctness.

Both the percentage of incomplete diagnoses and $E(\#N_d)$ increase as the cardinality of the actual fault set increases. The combined effect of such facts is responsible for the behavior shown in Figures 5 and 6, which report $\%suspect$, considering grids with $n=64$, $n=1024$ and $n=16384$. $\%suspect$ increases with the fault set cardinality, and the rate of increase is more sensible for large grids, indicating that the diagnosis does not converge toward asymptotic completeness.

The comparison of the performance of EDARS with grids of different degrees confirms the expected dependency of diagnosis correctness and completeness from the degree of the structure. This property is further evidenced in Figure 7, which plots the percentage of units which the algorithm is unable to diagnose as a function of the grid degree, considering fault sets of cardinalities ranging from $0.1n$, to $0.5n$. From the figure it is seen that the gain in diagnosis completeness due to the increased degree of the structure is more perceivable with large cardinalities of the fault set. For example, while grid G_3 is sufficient, on the average, to ensure almost complete diagnosis if the number of faults is $0.1n$, the degree of the grid should be at least 4 when this number is $0.3n$, and at least 6 with fault sets of cardinality $0.5n$.

6.2 Evaluation of EDARS with hypercubes

The most notable result of simulation experiments with hypercubes is the average of the syndrome-dependent bound T_σ , which is very close to n in the full range of simulation experiments, where fault set cardinalities ranged from $0.1n$ to $0.5n$. This means that the diagnosis returned by EDARS is almost certainly correct in all cases. Figure 8 displays $E(T_\sigma)$ expressed as percentage of the size, for hypercube structures H_6 , H_{10} and H_{14} . Contrary to the case of grids, this percentage tends to increase as the size of the structure increases, indicating that the diagnosis tends to converge toward asymptotic correctness.

For all the sizes considered, the diagnosis of hypercubes resulted always complete with fault sets of cardinality $0.1n$. Above this cardinality, simulation experiments reported a percentage of incomplete diagnoses which increase as fault sets become larger. However, the percentage of incomplete diagnoses was always below 40%, even with fault set of cardinality $0.5n$. When the diagnosis is incomplete, the average number of suspect units exceeded 1 only slightly, independently of the size of the hypercube. This means that, contrary to the case of grids, the percentage of suspect units in hypercubes tends to decrease as the size of the structure increases, i.e. the diagnosis tends to converge toward completeness as n tends to infinite. This behavior, displayed in Figure 9, is compared to the corresponding behavior of grids in Figure 10, assuming fault sets of cardinality $0.5n$. The fact that diagnosis provided by EDARS with hypercubes is asymptotically correct and complete agrees with the theoretical studies by [6] and [7], according to which a degree slightly above $\log n$ is necessary and sufficient to achieve asymptotically correct and complete diagnosis. Thus, EDARS may be regarded as a well-performing diagnosis algorithm for hypercubes and, reasonably, for regular structures in general.

6.3 Evaluation of EDARS with CCCs

The performance of EDARS with CCC structures was evaluated to the purpose of gaining insight on the influence of parameters of the interconnection structure, other than the degree, on diagnosis correctness and completeness. In fact, CCC structures have the same constant degree as triangular grids, but the diameter and the bisection width are sensibly different, and the differences may vary considerably with the size of the structure.

Simulation experiments considered CCCs of dimensions $d=4$, $d=7$ and $d=11$, corresponding to sizes $n=64$, $n=896$ and $n=22528$. Figure 11 reports $E(T_{\mathcal{O}})$ expressed as percentage of the size of the structure. It is seen that $E(T_{\mathcal{O}})$ is above $\#N_f$ in the entire range assumed for its cardinality, for sizes $n=64$ and 896 . For size $n=22528$, $E(T_{\mathcal{O}})$ was slightly less than $\#N_f$ when $\#N_f$ was $0.5n$, meaning that in this case the correctness of diagnosis cannot be guaranteed. However, simulation results showed that even in this case the diagnosis returned by EDARS was always correct. Contrary to the case of grids, the value of $E(T_{\mathcal{O}})/n$ increases as n increases as long as the cardinality of the fault sets is at most $0.4n$. The situation is reversed for fault sets of cardinality $0.5n$. The same behavior is displayed by $\%suspect$ (Figure 12), thus showing that the performance of EDARS with CCCs scales, to a certain extent, with the size of the structure.

6.4 Comparison of different structures

The influence of structure parameters other than degree over diagnosis correctness and completeness is enlightened by comparison of simulation results yielded by EDARS with structures of different diameter and/or bisection width. Figure 13 compares hypercube H_6 with grid G_6 of size $n=64$, and hypercube H_8 with grid G_8 of size $n=256$. As shown in Table 5, the degree and the diameter are the same for H_6 and G_6 and for H_8 and G_8 , while bisection widths are different, more noticeably in the case of G_8 and H_8 . Figure 13 plots the average number of suspect units, expressed as a percentage of the size. In G_6 and H_6 this percentage is essentially the same, except for a slightly superior performance of the hypercube when the cardinality of the fault set is relatively large. Comparing the same percentages obtained with structures of size 256, it is seen that H_8 tends to outperform G_8 as the cardinality of the actual fault set increases.

The different performances of EDARS with grids and hypercubes of the same size, degree and diameter, and the circumstance that diagnosis completeness increases more perceptibly in the case of H_8 and G_8 , where the difference of bisection widths is larger, support the conjecture that bisection width plays a role in determining the degree of diagnosis completeness.

This thesis is confirmed by comparing the simulation results obtained with grids G_3 and CCCs of approximately the same size, namely G_3 with $n=64$, 900 and 22500, and CCC_4 ($n=64$), CCC_7 ($n=896$) and CCC_{11} ($n=22528$). In this case, the degree is 3 for both grids and CCCs, but diameters and bisection widths are different, as seen from Table 6. As reported in Figures 14 and 15, CCC_7 outperforms grid G_3 of size $n=900$ in terms of diagnosis correctness and, more remarkably, in terms of diagnosis completeness. The advantage of CCC structure over G_3 increases with the increase of the fault set cardinality, although the results tend to converge as this cardinality approaches $0.5n$. A similar and more noticeable result comes from the comparison of CCC_{11} with G_3 of size $n=22500$. The preceding results agree with the circumstance that the diameter is smaller and the bisection width is larger than in the case of CCCs than in the case of G_3 , and that the differences are larger in the case of CCC_{11} and G_3 of size $n=22500$.

The comparison of CCC_4 with G_3 of size $n=64$ deserves special attention, because both structures have the same bisection width but, contrary to the situation occurring with larger sizes, the diameter of the grid is smaller than the diameter of the CCC. The expectation that EDARS performs better with G_3 than with CCC_4 is confirmed by simulation experiments. As seen from Figures 14 and 15, the performance with G_3 is slightly better than with CCC_4 in term of both diagnosis correctness and diagnosis completeness. The advantage of G_3 , is more noticeable in diagnosis completeness and it increases with the increase of the fault set cardinality.

Table 5. Degree, diameter and bisection width of hypercubes and grids G_6 and G_8 for some structure sizes.

	H_6	$G_6, n=64$	H_8	$G_8, n=256$
degree	6	6	8	8
diameter	6	6	8	8
bisection width	32	24	128	64

Table 6. Degree, diameter and bisection width of CCCs and grid G_3 for some structure sizes.

	$CCC_4, n=64$	$G_3, n=64$	$CCC_7, n=896$	$G_3, n=900$	$CCC_{11}, n=22528$	$G_3, n=22500$
degree	3	3	3	3	3	3
diameter	9	8	16	30	26	150
bisection width	8	8	64	30	1024	150

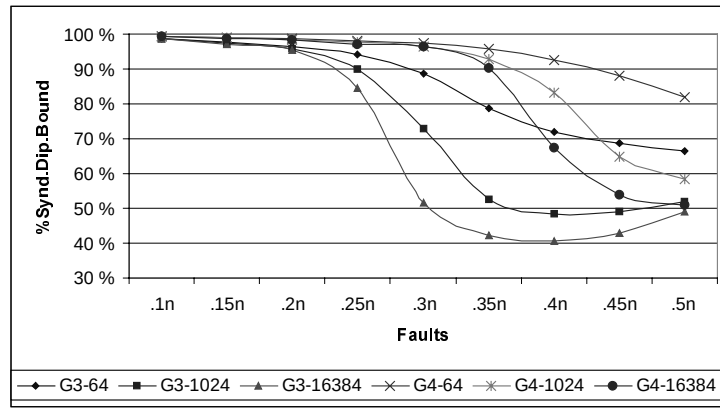


Figure 3. Triangular and square grids: value of $100E(T_{\mathcal{O}})/n$ for $n=64, 1024, 16384$.

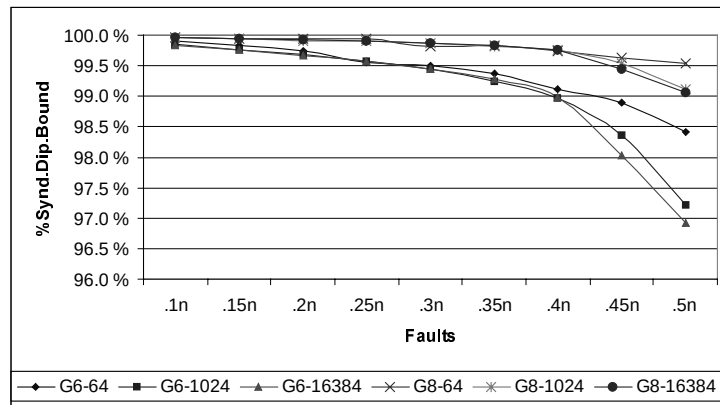


Figure 4. Hexagonal and octagonal grids: value of $100E(T_{\mathcal{O}})/n$ for $n=64, 1024, 16384$.

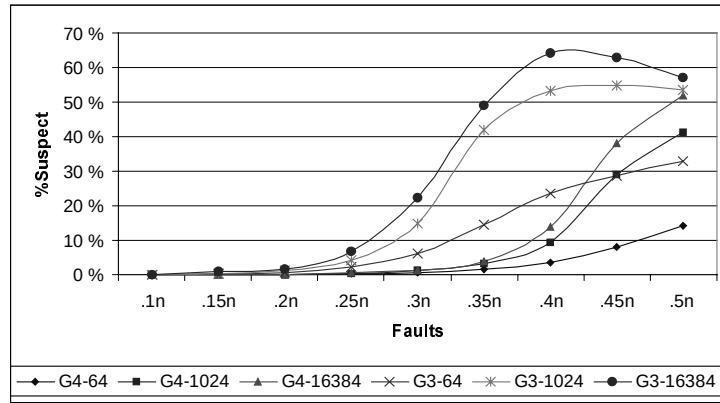


Figure 5. Triangular and square grids: percentage of suspect units for $n=64, 1024, 16384$.

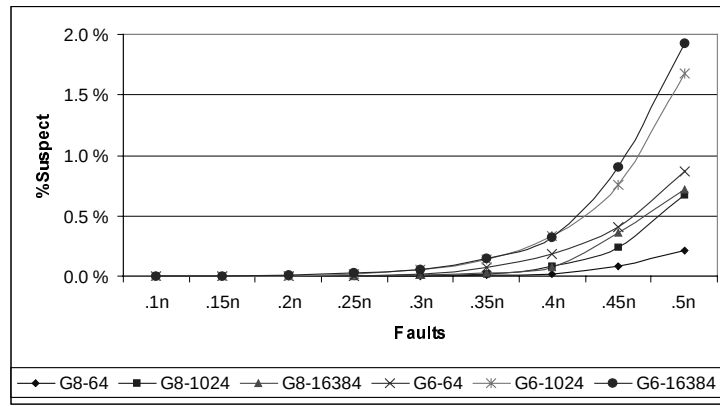


Figure 6. Hexagonal and octagonal grids: percentage of suspect units for $n=64, 1024, 16384$.

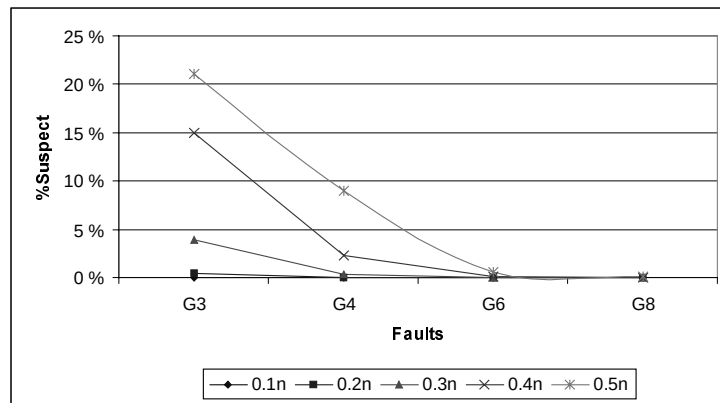


Figure 7. Percentage of suspect units for grids with $n=256$ and $\#N_f = 0.1n, 0.2n, 0.3n, 0.4n$ and $0.5n$.

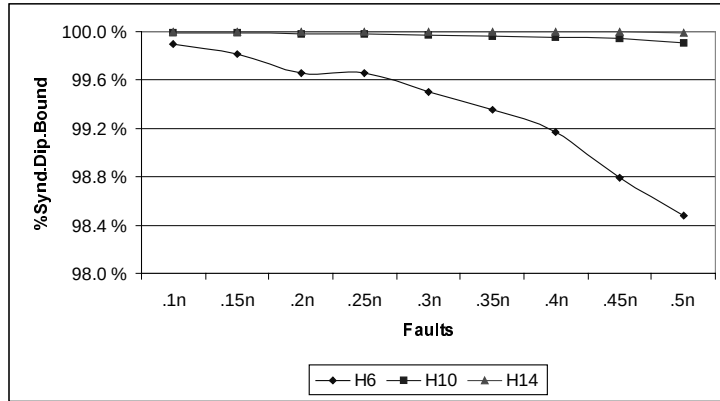


Figure 8. Hypercubes H_6 , H_{10} , H_{14} : value of $100E(T_\phi)/n$.

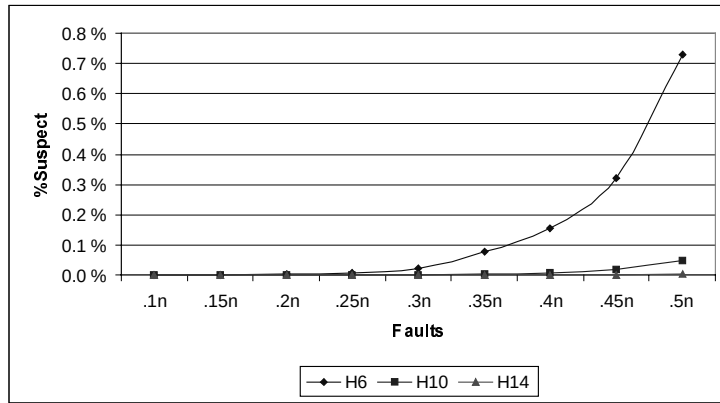


Figure 9. Hypercubes H_6 , H_{10} , H_{14} : percentage of suspect units.

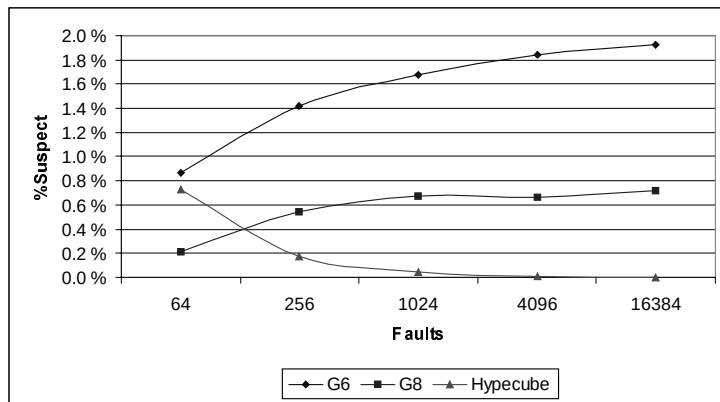


Figure 10. Percentage of suspect units for G_6 , G_8 and hypercubes with $\#N_f=0.5$.

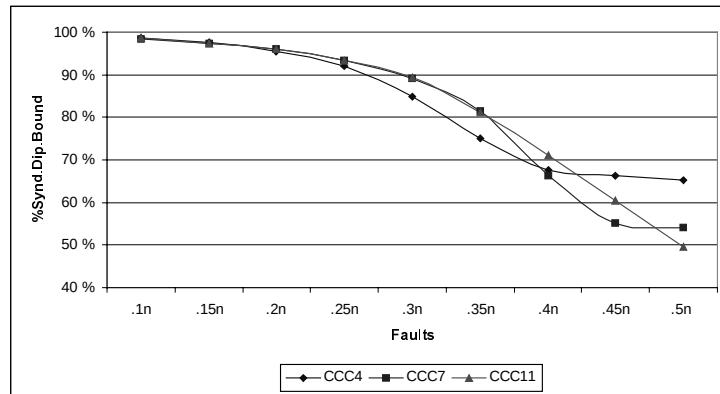


Figure 11. CCCs: value of $100E(T_{\mathcal{O}})/n$.

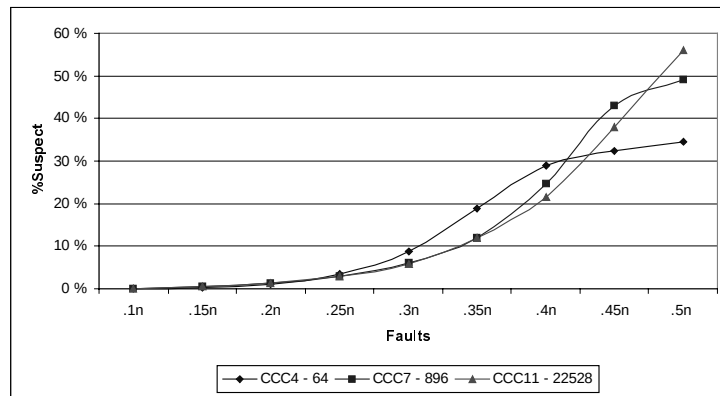


Figure 12. CCCs: percentage of suspect units.

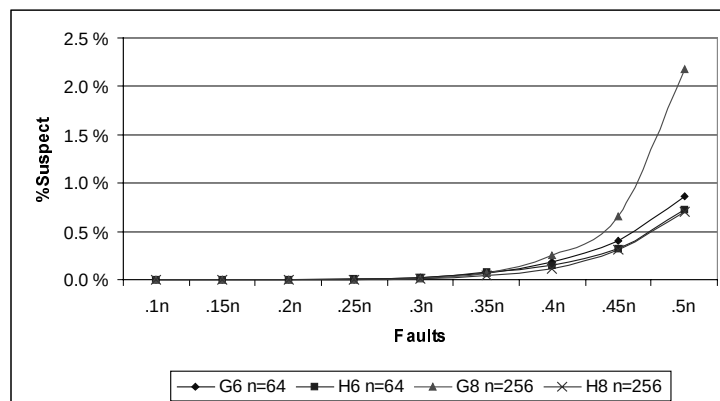


Figure 13. Percentage of suspect units for G_6 ($n=64$), H_6 , G_8 ($n=256$) and H_8 .

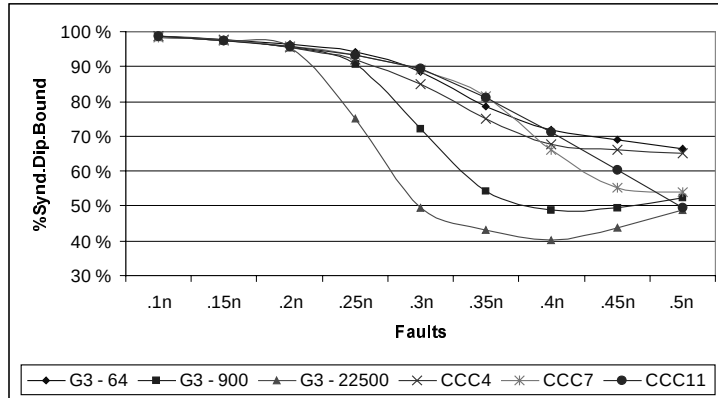


Figure 14. CCCs and triangular grids: value of $100E(T_\sigma)/n$.

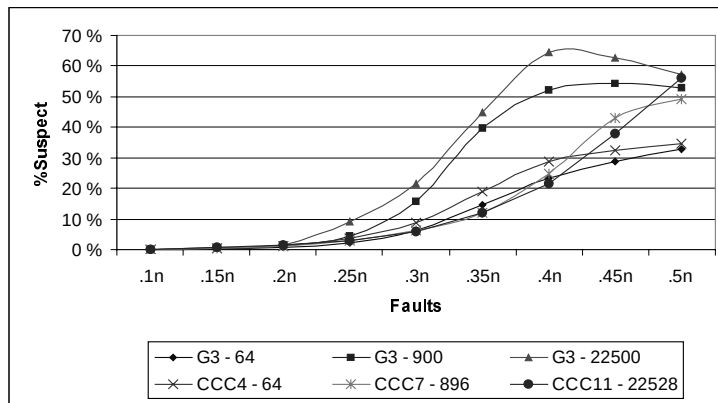


Figure 15. CCCs and triangular grids: percentage of suspect units.

7 Conclusion

An $O(nt)$ algorithm for the diagnosis of regular structures (EDARS) has been introduced. Given a syndrome σ , EDARS returns, along with the diagnosis, a syndrome-dependent bound T_σ . The diagnosis provided by EDARS is correct, but possibly incomplete, if the cardinality of the actual fault set is less than T_σ .

The performance of EDARS when applied to regular structures of different degrees, diameters and bisection widths has been evaluated by means of simulation.

The comparison between grids of different degrees, whose results are reported in Section 6.1, underlines the influence of degree over diagnosis correctness and completeness. From simulation experiments it is also seen that, regardless of the degree of the grid, the percentage of units that EDARS is unable to diagnose increases with the cardinality of the fault set, and that for given grid degree and fault set cardinality, this percentage increases as the size of the grid increases.

Comparisons of simulation results yielded by grids of selected sizes and degrees and by comparable hypercubes and CCCs, showed that, beside degree, diameter and bisection width also appear to influence the performance of EDARS, particularly with respect to diagnosis completeness.

The evidence that the degree of diagnosis completeness is notably influenced by some parameters of the interconnection structures other than the degree, is a relevant result, particularly in view of the application to wafer-scale testing, where almost complete diagnosis is a crucial goal. If this goal

could only be achieved by a relatively large degree, the cost of the interconnection structure to be embedded on the wafer would be negatively affected. The same goal could perhaps be achieved with reduced cost as a combined effect of multiple parameters. For this reason the extent to which diameter and bisection width influence the cost of the interconnection structure in a wafer environment deserves a deeper investigation. This way, cost/performance tradeoffs based on different choices of degree, diameter and bisection width could be evidenced and exploited in the identification of “good” interconnection structures for the purpose of wafer-scale testing.

8 References

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